Heuristic Approach for the Design of a High Availability Structure

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Summary

1. Introduction and motivation
2. Problem formulation
   - Two different single objective routing models aimed at:
     (i) the maximization of the average availability for Working Paths (WPs); (ii) the maximization of the average availability for all the flows (with the Backup Path (BP) on the spine or off the spine)
3. Heuristic resolution approach
   - Cost metric used in the spanning tree generation
   - Other heuristics for comparison
4. Some experimental results
5. Conclusions and further work
1. Introduction and Motivation

- The availability of the resources in communication networks is critical.
- Providing adequate levels of availability for every demand in a network is of paramount importance.
- For each demand, a pair of edge-disjoint paths is selected: the WP is used in regular conditions; in the event of failures in any of the components of the WP, a BP is used.
- This path pair provides end-to-end protection for critical service demands in the network.
1. Introduction and Motivation

• Considering the topological structure of a network, we select a set of links that provide a high availability path to be used by the different end-to-end demands.
• This set of links constitutes a high availability structure (the spine) and is used as the WP for each demand. In our approach, the spine is a spanning tree.
• The BP may use any link in the network including links in the spine. However, the WP and the BP must be edge-disjoint.
• The availability of the edges depends on whether they belong to the high availability structure \(a_{ON}\) or not \(a_{OFF}\) (similarly to previous work [1]).

The spine concept for improving network availability.
1. Introduction and Motivation

Previous work

• In [1] the BP should avoid edges of the spine, which is not required here. Allowing the BP to use edges of the spine requires a higher capacity of the links in the spine.

• Previous work [2] focuses on the optimization of the availability of the WP only (average and minimal). Different metrics for the enumeration of the spanning trees are proposed. The availability of the edges depends on their length and is taken into account in the selection of a suitable spine.


2. Problem Formulation

- We consider that the high availability structure (spine) should form a spanning tree. The total cost of the spanning tree is the sum of costs of the edges that form it.
- The availability of a path $p(s, t)$ between nodes $s$ and $t$ is the product of the availability of the edges that form it.
- Availability of a path pair:
  \[ A_{(s,t)} = 1 - \left( 1 - A_{WP(s,t)} \right) \left( 1 - A_{BP(s,t)} \right) \]
2. Problem Formulation

Problem 1

- **Objective:** maximization of the sum of the availabilities of the WPs of all the flows in the network (equivalent to the maximization of the average availability of the WPs of all the flows in the network).

- **Constraints to be considered:**
  - Flow conservation;
  - Edge-disjointness for the path pair;
  - Loop free routing;
  - The WPs should form a spanning tree.

- The function for calculating the availability of the WPs is linearized. An ILP model is formulated.
2. Problem Formulation

Problem 1

• As the availability of the edges in the spine is fixed \( a_{ON} \), finding the most available WP for each demand is equivalent to finding the WP with the smallest number of hops in the spine for each demand. \( \Rightarrow \) Problem of finding a spanning tree with the smallest number of hops for the paths in that spanning tree.

• Afterwards: application of Dijkstra algorithm to find an edge-disjoint BP for each demand, where the BP (i) may or (ii) may not use edges of the spine.
2. Problem Formulation

Problem 2

- Objective: maximization of the sum of the availabilities of all the flows in the network (equivalent to the maximization of the average availability of all the flows in the network).
- Similar constraints to the ones in Problem 1, plus constraints for calculation of the availability for each flow.
- Need for linearization of the expressions involving the product of variables.
- Two variations of the problem are formulated, regarding whether the BP (i) may or (ii) may not use edges of the spine.
3. Heuristic Resolution Approach

• As we are dealing with an unweighted graph, we consider the length of each edge as 1.

• Let $\sigma_k(s, t) = |P_k(s, t)|$, where $P_k(s, t)$ is the set of paths between $s$ and $t$, whose length is not higher than the shortest path length $L_0$ plus $k$ (non-negative integer).

• Let $\sigma_k(s, t | l)$ be the number of paths in $P_k(s, t)$ which include edge $l$.

• Topological structural measure: $k$-betweenness centrality for edge $l$ [3]:

$$B_k(l) = \sum_{\forall s,t: s \neq t} \frac{\sigma_k(s, t | l)}{\sigma_k(s, t)}$$

Generalized $k$-betweenness centrality using short paths and a parallel multithread implementation.
3. Heuristic Resolution Approach

- Centrality cost of each edge:
  \[
  c_k(l) = -B_k(l) + \max_{\forall \ell} B_k(\ell) + 1
  \]
- The cost of an edge is higher if it is less central (i.e. it appears in a smaller number of short paths).
- Heuristic based on the calculation of spanning trees (using Prim’s algorithm) minimizing the total centrality cost and avoiding certain edges (to diversify the obtained trees).
- For a \((s, t)\) demand for which no edge-disjoint BP exists, consider \(s\) or \(t\) (the one with higher average centrality) and consider avoiding one of the edges in the spine passing in this node, for a certain number of iterations.
- Average centrality cost for a node: average value of \(c_k(l)\) for all edges \(l\) leaving or entering the node.
3. Heuristic Resolution Approach

**Input:** $k$; $maxIter$.

**Output:** Feasible spines from which the one with best $A_{aWP}$ and the one with best $A_a$ may be identified.

```plaintext
for listReset = 0 to 1 do
    loop
        if listReset == 1 then
            Reset the current list of edges to be avoided.
        end if
        The first time this inner loop is run, no new edges to be avoided are defined; the following $|L|$ times, one edge at a time should be avoided, with the edges being selected in decreasing order of centrality cost (i.e. the least central edges are added first).
```

DRCN 2019
3. Heuristic Resolution Approach

repeat
  Calculate spine (with Prim’s algorithm) that tries to avoid edges on the list and minimizes the total centrality cost, with centrality cost of each edge $c_k(l)$.
  
  if spine is feasible then
  Calculate BPs for every demand.
  Calculate performance measures.
  
  else
  Identify a demand $(s, t)$ for which no disjoint BP exists.
  Considering $s$ and $t$, select the node with higher average centrality.
  The edge (or one of the edges) in the spine passing in this node should be avoided for $maxIter$ iterations.
  Update list of edges to be avoided: decrease the number of iterations during which they should be avoided.
  
  end if
  
  until a feasible spine is found.
end loop / end for
3. Other Heuristic Resolution Approaches for Comparison

• In [1], structural properties of a network topology are used to select a suitable spine. The edge cost is a weighted sum of the edge degree and the edge betweenness.
• A minimum cost spanning tree is calculated.
• If an edge disjoint BP cannot be obtained for a node pair, the set of conflicting edges (those shared by a WP and the maximally link disjoint corresponding BP) are identified.
• A new spanning tree is obtained avoiding each of those links (and all possible combination of them).
• Among the admissible spanning trees the one with best performance according to the relevant metric is saved.

3. Other Heuristic Resolution Approaches for Comparison

• In [2], a heuristic based on the enumeration of candidate spines (spanning trees) by non-decreasing order of the total cost is proposed.

• The cost metric for each edge is $c_k(l)$.

• A total of $|N| \cdot |L|$ trees is obtained, of which some may be infeasible due to not having edge-disjoint path pairs for all the demands.

4. Experimental Results

• Results for two networks

| Network | $|\mathcal{N}|$ | $|\mathcal{L}|$ | $\mu$ | $|\mathcal{N}| \cdot |\mathcal{L}|$ | $\delta$ |
|---------|---------------|---------------|-------|-------------------------------|--------|
| polska  | 12            | 18            | 3.00  | 216                           | 4      |
| italia  | 32            | 69            | 4.31  | 2208                          | 6      |

• Exact results were obtained for problem 1 (polska and italia) and for problem 2 (polska) where the BP (i) may or (ii) may not use edges of the spine.
• No distinction is made between alternative optimal solutions.
4. Experimental Results

- Experiments with the heuristic using the cost metric $c_k(l)$ were performed for $k = 0, 1, 2$.
- An increase in $k$ means that a larger number of shortest paths is taken into consideration in the calculation of the cost metric.
- According to the conclusions in [2], the value of $k$ does not have to be very high, and it is enough to find the shortest paths and the paths with length close to these ones.

### 4. Experimental Results

#### TABLE II: Results for the polska network

**(a) BP may use edges of the spine**

<table>
<thead>
<tr>
<th>Method</th>
<th>$k$</th>
<th>$\max \text{Iter}$</th>
<th>$A_a$</th>
<th>$A_{a WP}$</th>
<th>$h_S$</th>
<th>$d_{iS}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Exact</td>
<td></td>
<td></td>
<td>0.9999487</td>
<td>0.99734</td>
<td>2.6667</td>
<td>5</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>0.9999566</td>
<td>0.99660</td>
<td>3.4091</td>
<td>8</td>
</tr>
<tr>
<td>Heuristic in [14]</td>
<td></td>
<td></td>
<td>0.9999347</td>
<td>0.99734</td>
<td>2.6667</td>
<td>5</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>0.9999560</td>
<td>0.99658</td>
<td>3.4242</td>
<td>8</td>
</tr>
<tr>
<td>Heuristic in [20]</td>
<td>2</td>
<td>1,2</td>
<td>0.9999488</td>
<td>0.99734</td>
<td>2.6667</td>
<td>5</td>
</tr>
<tr>
<td></td>
<td>1,2</td>
<td>≥3</td>
<td>0.9999534</td>
<td>0.99720</td>
<td>2.8030</td>
<td>6</td>
</tr>
<tr>
<td>Proposed heuristic</td>
<td>1,2</td>
<td>0</td>
<td>0.9999488</td>
<td>0.99734</td>
<td>2.6767</td>
<td>5</td>
</tr>
<tr>
<td></td>
<td>0</td>
<td>≥3</td>
<td>0.9999531</td>
<td>0.99728</td>
<td>2.7272</td>
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</tbody>
</table>

**(b) BP should avoid edges of the spine**

<table>
<thead>
<tr>
<th>Method</th>
<th>$k$</th>
<th>$\max \text{Iter}$</th>
<th>$A_a$</th>
<th>$A_{a WP}$</th>
<th>$h_S$</th>
<th>$d_{iS}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Exact</td>
<td></td>
<td></td>
<td>0.9999289</td>
<td>0.99734</td>
<td>2.6667</td>
<td>5</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>0.9999480</td>
<td>0.99660</td>
<td>3.4091</td>
<td>8</td>
</tr>
<tr>
<td>Heuristic in [14]</td>
<td></td>
<td></td>
<td>0.9999294</td>
<td>0.99734</td>
<td>2.6667</td>
<td>5</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>0.9999480</td>
<td>0.99660</td>
<td>3.4091</td>
<td>8</td>
</tr>
<tr>
<td>Heuristic in [20]</td>
<td>2</td>
<td>0</td>
<td>0.9999322</td>
<td>0.99734</td>
<td>2.6667</td>
<td>5</td>
</tr>
<tr>
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<td>0</td>
<td>2</td>
<td>0.9999480</td>
<td>0.99660</td>
<td>3.4091</td>
<td>8</td>
</tr>
<tr>
<td>Proposed heuristic</td>
<td>2</td>
<td>0</td>
<td>0.9999322</td>
<td>0.99734</td>
<td>2.6667</td>
<td>5</td>
</tr>
<tr>
<td></td>
<td>0</td>
<td>2</td>
<td>0.9999480</td>
<td>0.99660</td>
<td>3.4091</td>
<td>8</td>
</tr>
</tbody>
</table>
4. Experimental Results

**TABLE III: Results for the italia network**

(a) BP may use edges of the spine

<table>
<thead>
<tr>
<th>Method</th>
<th>$k$</th>
<th>$max\ Iter$</th>
<th>$A_a$</th>
<th>$A_{a WP}^{WP}$</th>
<th>$h_S$</th>
<th>$d_{iS}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Exact</td>
<td></td>
<td></td>
<td>0.9999137</td>
<td>0.99643</td>
<td>3.5726</td>
<td>6</td>
</tr>
<tr>
<td>Heuristic in [14]</td>
<td></td>
<td></td>
<td>0.9998982</td>
<td>0.99616</td>
<td>3.8468</td>
<td>8</td>
</tr>
<tr>
<td>Heuristic in [20]</td>
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<td></td>
<td>0.99999003</td>
<td>0.99545</td>
<td>4.5605</td>
<td>10</td>
</tr>
<tr>
<td>0</td>
<td></td>
<td>5</td>
<td>0.9999164</td>
<td>0.99620</td>
<td>3.8085</td>
<td>8</td>
</tr>
<tr>
<td>0</td>
<td>1</td>
<td>5</td>
<td>0.9999180</td>
<td>0.99598</td>
<td>4.0262</td>
<td>9</td>
</tr>
<tr>
<td>0</td>
<td>0</td>
<td></td>
<td>0.9999139</td>
<td>0.99619</td>
<td>3.8185</td>
<td>7</td>
</tr>
<tr>
<td>0</td>
<td>0</td>
<td></td>
<td>0.99999194</td>
<td>0.99600</td>
<td>4.0040</td>
<td>9</td>
</tr>
</tbody>
</table>

(b) BP should avoid edges of the spine

<table>
<thead>
<tr>
<th>Method</th>
<th>$k$</th>
<th>$max\ Iter$</th>
<th>$A_a$</th>
<th>$A_{a WP}^{WP}$</th>
<th>$h_S$</th>
<th>$d_{iS}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Exact</td>
<td></td>
<td></td>
<td>0.9998297</td>
<td>0.99643</td>
<td>3.5726</td>
<td>6</td>
</tr>
<tr>
<td>Heuristic in [14]</td>
<td></td>
<td></td>
<td>0.9998015</td>
<td>0.99623</td>
<td>3.7742</td>
<td>8</td>
</tr>
<tr>
<td>Heuristic in [20]</td>
<td>1</td>
<td></td>
<td>0.9998115</td>
<td>0.99623</td>
<td>3.7742</td>
<td>8</td>
</tr>
<tr>
<td>2</td>
<td></td>
<td></td>
<td>0.9998394</td>
<td>0.99620</td>
<td>3.8085</td>
<td>8</td>
</tr>
<tr>
<td>0</td>
<td>1</td>
<td>17,18</td>
<td>0.9998146</td>
<td>0.99619</td>
<td>3.8185</td>
<td>7</td>
</tr>
<tr>
<td>0</td>
<td>0</td>
<td></td>
<td>0.9998392</td>
<td>0.99567</td>
<td>4.3427</td>
<td>10</td>
</tr>
</tbody>
</table>
4. Experimental Results

- Information on the tables: availability values $A_a$ and $A_a^{WP}$ obtained in the experiments considering the exact formulations and the heuristics; $h_s$, average number of hops in the spine; $d_i$, spine diameter.

- No distinction is made between alternative optimal solutions. Example: different spines with the same value of $A_a^{WP}$ may lead to different solutions in terms of BPs and $A_a$.

- Execution times:
  - Exact resolution – polska: a few seconds for problem 1 and a few hours for problem 2; italia: a few days for problem 1.
  - Heuristics – a few seconds for any network, except heuristic in [2], which takes about 10 minutes for the Italia network.

4. Experimental Results

- The spines leading to the best solution in terms of $A_a$ present a higher diameter and a higher average number of hops of the WPs. The spine does not include mainly the central edges.
- The spines leading to the best solution in terms of $A_a^{WP}$ tend to include mainly the central edges. They present a lower diameter and a lower average number of hops of the WPs.
- The maximization of the WP availability is not closely related to the maximization of the path pair availability. In fact, for the best solution in terms of $A_a$, the average number of hops of the WPs is higher and consequently the average availability of the WPs is lower. This observation justifies the need to study the problems regarding both parameters and not just $A_a^{WP}$. 
4. Experimental Results

(a) Exact solution for problem 1

(b) Solution that minimizes the total centrality cost, calculated for $k = 2$ (which is also the first admissible spine)
4. Experimental Results

• The exact solution that minimizes $A_{a}^{WP}$ (see Fig. (a)) mainly includes more central edges, as expected, as this type of solution tends to lead to a set of shorter (in hop count) and more available WPs in the spine.

• The minimal cost admissible solution obtained considering the edge cost $c_{k}(l)$ with $k = 2$ (see Fig. (b)) presents some very long and apparently not central edges. Although in topological terms, these edges do not seem to be very central, they are used in many min-hop paths, and so they will have a low centrality cost, given the definition of edge cost $c_{k}(l)$. 
4. Experimental Results

- The best values of $A^WP_a$ were usually obtained with $k = 2$, whereas the best values of $A_a$ were usually obtained with $k = 0$. This has to do with the nature of the centrality measure considered in this study.

- By considering higher $k$, i.e. longer paths in the calculation of the metric $B_k(l)$ (used in the spanning tree calculation), the measure of centrality of the edges becomes more accurate. By identifying the more central edges, the obtained spines tend to focus on those edges, originating trees with smaller diameter.
4. Experimental Results

• Comparing the results with different heuristics:
  – polska, allowing the BPs to use edges of the spine: the exact solution and the solution with heuristic in [1] present a better value for $A_a$, at the expense of longer WPs (trade-off).
  – italia, allowing the BPs to use edges of the spine: the proposed heuristic finds the best solution for $A_a$, without incurring in longer WPs.
  – italia: no heuristic found a solution with the optimal exact value of $A_a^{WP}$ (see Fig. (a)).

5. Conclusions

• The proposed heuristic (based on spanning tree calculation with some edges being avoided) leads to solutions with high availability.

• The considered centrality measure used for calculation of the costs $c_k(l)$ proved to be appropriate, leading to solutions with high availability $A_a$.

• Future work:
  – other performance measures related to the minimal availability of the WPs or the path pairs;
  – turning the proposed heuristic into a tabu search procedure, with some degree of randomness in the selection of edges to avoid;
  – considering information on traffic demands and edge capacities;
  – other spine configurations (not necessarily a spanning tree).
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